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NEW THEORETICAL PERSPECTIVES ON THE DISTRIBUTION OF INCOME AND WEALTH AMONG INDIVIDUALS: PART II: EQUILIBRIUM WEALTH DISTRIBUTIONS

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ABSTRACT

This paper investigates the determination of the equilibrium distribution of income and wealth among individuals within a simple equilibrium growth model, where there is consistency between the movements of aggregate variables and the savings, bequest, and reproduction behavior of individuals. It describes centrifugal and centripetal forces, (leading to more or less unequal distributions), identifies the factors that may have contributed to the observed increase in inequality, and provides explicit expressions for the level of tail-inequality in terms of the underlying parameters of the economy and policy variables.

Among the key results are: (i) The magnitude of wealth inequality does not, in general depend on the difference between the rate of interest (r) and the rate of growth (g); the former is itself an endogenous variable that needs to be explained. In the standard generalization of the Solow model, in the long run not only is $r < g$, but $sr < g$ (where s is the savings rate). (ii) An increase in capital taxation may be (and in some of the central models is) fully shifted, and so may not lead to lower levels of inequality. (iii) If the capital tax is progressive and/or the proceeds go to public investment, wealth inequality may be reduced the well-being of workers may be increased.

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Introduction

In Part I of this paper, we described a number of "new" stylized facts—broad economic regularities that seemed *different* from those characterizing mid twentieth century economies. We observed that there has been an increase in income and wealth inequality.

Piketty (2014) suggested that since the rate of return on capital exceeded the rate of growth of the economy, capitalists' wealth would grow relative to the size of the economy, and since the return to capital had not declined significantly, that meant an ever and ever increasing share going to capitalists.

While earlier work (Stiglitz, 1966, 1969a) had identified some circumstances in which the economy could be characterized by wealth divergence, more typically, the economy converges to an equilibrium wealth distribution (Bevan, 1978, Stiglitz, 1966, 1969a, 1978, Bevan and Stiglitz, 1979). There are forces in the economy which lead to *increases* and *decreases* in wealth inequality. We can think of these as centrifugal and centripetal forces. There exists an equilibrium distribution of wealth when the two sets of forces are balanced.

This paper is conducted within a neoclassical framework, where output depends on capital and labor through a constant returns to scale production function. In this framework, an increase in the capital output ratio necessarily is associated with a decrease in the return to capital in the long run. Our models explicitly require micro- and macro-consistency: increased accumulation leads to lower returns, until the condition that the rate of return is greater than the rate of growth is no longer satisfied. We explore two variants of the standard model. In the a generalization of the standard Solow model in which all individuals save at the same rate (or in an extension, where they all have the same savings function), not only is r, the return on capital, less than g, the rate of growth, but even $sr < g$, where s is the savings rate.

Some have criticized Piketty for simply assuming that capitalists save everything. We extend the model to allow them to have a savings rate less than unity. The long run effect is to increase the rate of return in a fully offsetting way. In the Kaldor model, where capitalists save a fraction s_p of their income, and workers save nothing, then $s_p r = g$ in the long run, so even though r exceeds g, the wealth of the capitalists increases at the same rate of the economy. In the absence of stochastic returns, there is no tendency for ever increasing inequality.

But for precisely the same reason, the suggestion that to reduce wealth inequality, one should tax capital fails: for the tax is fully shifted.

Using these long run equilibrium conditions allows us to extend the earlier studies of equilibrium wealth distributions[2], deriving new closed form solutions described the tails of the wealth distributions. The paper is divided into 3 parts (besides this introduction and a conclusion). In section 1 we set up the basic model, a variant of the Solow model. In section 2, extend the analysis to a Kaldorian (1957) model which captures much of Piketty's analysis. In the simplistic version of the Kaldor model explored here, workers save nothing. In a more realistic version, they save for their retirement. In Part III of this paper, we explore that extension, showing that we obtain results similar to those obtained here, but at the same time we are able to obtain a simple expression relating the ratio of life cycle wealth to capitalists' wealth. Again, contrary to Piketty's suggestion, there is a long run equilibrium: their wealth does *not* continuously increase relative to that of workers. In section 3 we use these models to discuss the centripetal and centrifugal forces—to identify changes that might account for the increased inequality.

1. Basic Model: Generalizing Solow to Heterogeneous Dynastic Families

The basic model is a variant of the Solow growth model, where we think of the economy as consisting of dynastic families, leaving equal bequests among their children. For simplicity, we initially ignore technical change. The evolution of wealth per capita for the ith family is described by the differential equation

(1.1) $\frac{d}{dt}(\log k_i) = s_i y_i - n_i$,

where y_i is the ith family's income (per capita)

(1.2.) $y_i = w_i + r_i k_i$,

where w_i is the ith family's wage, r_i is its return on capital, and k_i is its capital (per capita). We assume that there is perfect inheritance of both labor market and capital market productivity. n_i is the ith family's rate of reproduction.[3]

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[2] This section also incorporates previously unpublished results in Stiglitz (1966)

[3] If N_i is the size of the *i*th family, $\frac{dlog(N_i)}{dt} = n_i$. $k_i = \frac{K_i}{N_i}$, so that $\frac{dlog(k_i)}{dt} = \frac{dlog(K_i)}{dt} - n_i$. $k = \frac{\sum K_i}{\sum N_i} = \sum q_i k_i$ where $q_i = \frac{N_i}{N}$, the proportion of the population in the *i*th family.

An essential part of the analysis is macro- and micro- consistency: aggregate k (the aggregate capital labor ratio[4]) determines the average return on capital, r, and wages with

(1.3) $K = \sum K_i$

where K_i is the ith family's total capital stock, K is the aggregate capital stock of the economy, and k = K/L, where k is the aggregate capital labor ratio and L is the labor supply. We assume a neoclassical production function where output per worker is $f(k)$, and

(1.4) $r_i = \rho_i f'(k)$

(1.5) $w_i = v_i(f(k) - f'(k)k)$,

where ρ_i is the *relative* return to the ith family's investment (some families are able to obtain a higher return from their investments than others, with $f'(k)$ being the *average* marginal return across all families)[5] and where v_i is the *relative* return to the ith family's labor (some families receive higher wages—payments per unit labor--than others, with $f(k) - f'(k)k$ being the *average* wage across all families).

1.1 The Basic Solow Model

Consider a Solow model, where a constant fraction of income, s, is saved, and w, s, r, and n are the same for all families. Then

(1.6) $\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j) = sw\left(\frac{1}{k_i} - \frac{1}{k_j}\right)$,

There is convergence of the wealth distribution: **regardless of initial distribution of wealth, there will eventually be equality of wealth.** Notice that this result holds no matter what the value of s and how it is determined.

Note that in the long run equilibrium, $sf(k) = nk$, or $\frac{sf(k)}{k} = \frac{sw}{k} + sr = n$, so sr is always less than n, the rate of growth. While Piketty, in his analysis, emphasized the relationship between the rate of return and the rate of growth (with capital growing faster than income if the rate of return exceeded the rate of growth), it is clear that what matters is not the rate of return, r, but sr; and we have shown that,

[4] When, later, we introduce labor augmenting technological change, k will stand for the capital-effective labor ratio.

[5] That is, in the obvious notation, $E(\rho) = 1, E(v) = 1$

in the long run, in the standard Solow growth model, sr *must* be less than the rate of growth, not greater, as he hypothesized. (We will return to this later, in the context of other growth models.)

If s, r, and n are the same, but w_i differs across families, then in steady state the wealth distribution corresponds precisely to the wage distribution. Asymptotically,

(1.7) $\frac{k_i^*}{k_j^*} = \frac{w_i}{w_j}$.

If wages are lognormally distributed, so will wealth.[6]

1.2 Extension to technological change

The model can be extended to labor augmenting technological change, where now k denotes the capital stock per effective worker, K/La, where a is the number of efficiency units associated with any worker (at a given time). Then, instead of (1.5) we have

(1.5') $w_i = av_i(f(k) - f'(k)k)$.

Nothing else in the analysis changes. Now the rate of growth of the economy, g, is given by

(1.8) $g = (1 - S_k)(n + \lambda) + S_k sf(k)/k$,

where λ is the rate of labor augmenting technological change, equal to $\frac{dlog(a)}{dt}$ and where S_K is the share of capital. (By constant returns to scale, the sum of the shares of capital and labor equal unity.) In the long run

(1.8') $g^* = n + \lambda = sf(k^*)/k^*$

Since none of this affects (1.6), it is still true that, under the stipulated conditions, the distribution of wealth depends only on the distribution of wages. In the long run, neither r, nor g, nor the difference between r and g, has any impact. Indeed, looking more closely at (1.6), we see that not even the speed of convergence depends on r, g, or $r - g$.

Later models will show that these results are quite general.

1.2. **Generalized Savings Functions**

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[6] In Stiglitz (1969a) we describe an array of other possibilities, including divergent paths. Here, we focus on what we consider to be the central cases. See also the discussion below.

Critical to the convergence result was the assumption that the savings rate, the rate of return to capital, and the reproduction rate do not depend on income (i.e. if individuals all receive the same wage, on k_i). Indeed, if that is the case, even with identical individuals (families), there can exist multiple steady states, i.e. multiple values of k for which s(k)f(k) = n(k)k. Figure 1 shows a case where the savings function is such that the savings rate is low for low incomes, and then the marginal savings rate increases at a critical threshold, y^, with the corresponding value of k^ (y^ = f(k^)). As the figure illustrates, there can be a low level equilibrium k*, where the savings rate is low, and another stable one k** where the savings rate is high. [7]

Similar results hold if, above a certain level of income, the rate of reproduction (rate of growth of labor supply) falls.

But under these circumstances, not only can the economy be trapped in a low equilibrium, so can families. Assume the economy is in steady state, with a given r. Assume the savings function of each family is a function of its income, i.e. of its k_i. Then, instead of (1.6), we have[8]

$\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j) = w\left(\frac{s(k_i)}{k_i} - \frac{s(k_j)}{k_j}\right) + r\left(s(k_i) - s(k_j)\right)$. For the existence of multiple equilibria, we require s' to be sufficiently large for some values of k_i : $\frac{dlog(s)}{dlog(k)} > (1 - S_k)$.

Now, there are an infinity of possible equilibria. Assume, for instance, that there are only two groups in the population, with a proportion of the population in the high wealth group being denoted by γ. Then $\{k_H, k_L, k, \gamma\}$ constitute an equilibrium if

(1.6') $d \ln k_i/dt = ws(k_i)/k_i + rs(k_i) - n = 0$

(1.9) $k = \gamma k_H + (1 - \gamma)k_L$

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[7] For there to be more than one equilibrium, $d\ln s/d\ln k + S_K - d\ln n/d\ln k > 1$ for some values of k. (Note that this inequality can never be satisfied if s and n are constant).
A perhaps more natural formulation would have s depend on income, and income depend on k, i.e. s(y(k)), so $d\ln s/d\ln k = (d\ln s/d\ln y)S_k$.
Not all the solutions to s(k)f(k)=n(k)k are stable in the natural sense, i.e. a slight increase in k can lead to an increase in savings that leads to further increases in k. Stability requires sf(k)/n(k) to cross the 45 degree line from above.

[8] The family's savings function (expresses as a function of k) is slightly different from that described earlier, since each family takes r and w as fixed. Hence, writing again savings as a function of income, y_i, $s(w + r k_i)$, and $d\ln s/d\ln k_i = (d\ln s/d\ln y_i) S_{ki}$, where $S_{ki} = rk_i/y_i$, the share of the family's income derived from capital, which as k_i increases, approaches unity. The critical value of dln s/dln y such that there exists multiple equilibra is thus lower, i.e., there might exist an equilibrium wealth distribution even under conditions under which there might not exist multiple steady states.

Thus, for any value of k, we can find the (stable) values of k_i satisfying (1.6'). There is then a value of γ which generates that value of k through (1.9).

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Poverty Traps with more generalized savings functions

In the years before the 2008 crisis, the bottom 80% consumed more than their income. Their savings function might be approximated by

(1.10a) $\boldsymbol{S_i} = -m + sy_i$, for $k_i > 0$ or $sy_i > m$

(1.10b) $\boldsymbol{S_i} = 0$ if $k_i = 0$ and $sy_i < m$

where $\boldsymbol{S_i}$ represents savings. (1.10b) says that when individuals have no wealth to dissave, there is no more dissaving: individuals cannot borrow.[9] Note that there is a discontinuity in the savings function at $k_i = 0$. Now (1.6) becomes

(1.6') $\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j) = (sw - m)\left(\frac{1}{k_i} - \frac{1}{k_j}\right)$, for $k_i, k_j > 0$,

so we get convergence so long as $sw > m$, i.e. so long as there is net savings out of wages.

But even if $sw < m$, there *can be* a long run macro-economic equilibrium, where a proportion of the workers, say $1 - \gamma$, are trapped in a zero wealth equilibrium, saving nothing, and all capitalists have the same wealth. Then, in steady state

(1.11) $\frac{\gamma(sw(k^*) - m)}{k^*} + sr - (n + \lambda) = 0.$

The steady state depends on γ , and this in turn depends on history: history matters. But the equilibrium described by (1.11) (for any given value of γ) is unstable. Returning to (1.6'), we see that if all the capitalists have the same wealth, then the "equality among capitalists" equilibrium can be sustained. But if any capitalist gets more than any other, his wealth continues to grow without bound relative to the other. (Later in this section, we will describe the possibility of a more stable low wealth-trap.)

[9] It represents a tight borrowing constraint. It is easy to generalize (1.10a) to include cases where there is a limit to (net) indebtedness.

The steady state equilibrium with $sw < m$ is also unstable in a macro-sense. If k increases to a value slightly above k^*, k continues to increase, eventually resulting in a high wage equilibrium, where $sw > m$, and all the convergent results described earlier apply.[10]

1.3. Wealth begets wealth

We have assumed so far that all individuals receive the same return on capital. But it is possible that the return that the ith family receives on its investment is an increasing function of its wealth: $r_i = r(k_i)$.[11] [12]

Returns to investment are related to the investments one makes in acquiring information, and the optimal investment in information is a function of the size of one's wealth (simply because information is a fixed cost.) Assume all families have the same wage, savings and reproduction rates. Again, let the proportion of the population in the high wealth group be denoted by γ. Then $\{k_H, k_L, k, \gamma\}$ constitute an equilibrium if

(1.6") $d \ln k_i/dt = ws/k_i + sr(k_i) - n = 0$

(1.9) $k = \gamma k_H + (1 - \gamma)k_L$

Thus, for any value of k, we can again find the (stable) values of k_i satisfying (1.6"). There is then a value of γ which generates that value of k through (1.9).

1.4. Extension to stochastic models

The stark results derived so far depend greatly on the absence of stochasticity in the economy. If individuals randomly have high wages and high returns and pass some of that good fortune to their children, their children will be wealthier. While the convergent process described in the basic model ensures that the effect of such good fortune attenuates over time, even as it does, new stochastic events create new equality. The equilibrium wealth distribution is thus a balancing out of these two

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[10] The upper equilibrium is the high k solution to $\frac{sw(k^{**})-m}{k^{**}} + sr(k^{**}) - (n + \lambda) = 0.$

[11] It is not inevitable that there be such a relationship: Though information is a fixed cost, one can imagine a financial market which amortized these fixed costs uniformly, in which case those with large amounts of wealth would obtain the same returns as those with lesser amounts of wealth (putting aside the slight differences that might arise from non-linear transactions costs.) But, again, because of imperfections of information, there are costly agency problems, and those with enough wealth may be able to mitigate these agency problems by running their own investment fund. As we comment later, the markets for information may be an important driver in current inequalities.

[12] Some of the higher average returns of the wealthier may be a result of their being better able to bear risk.

effects. In the following paragraphs we show how straightforward extensions of our model not only leads to an equilibrium wealth distribution, but one for which we can calculate the asymptotic magnitude of the tail inequality. In doing so, we continue to make use of the conditions ensuring consistency between the micro- and macro-variables and the implied conditions for long run equilibrium.

1.4.1 *Variable returns to capital*[13]

Assume now that the return to capital of each family is stochastic, with an i.i.d. distribution. For simplicity, here (and in most of the rest of this paper) we assume there is no labor augmenting technological progress (so $\lambda = 0$), which in turn implies that the long run growth rate g* is just the rate of growth of the population, n.

Then, even if there were no wage inequality, there would be wealth inequality. Families that had a run of good luck--high values of r--would have accumulated far more wealth than those who had bad luck. For these rich individuals, wages become negligible, and for these individuals, the wealth accumulation equation can be approximated by[14] [15]

(1.1") $dk_t = (sw - \mu k_t)dt + \sigma k_t dZ_t$

where the risk is associated with the return on capital and is proportional to sr:

(1.12) $$\sigma = sr\bar{\sigma}$$

and where μ is the drift in the stochastic process

(1.13) $$\mu = n - sr > 0.$$

Then it is standard that the stationary wealth distribution has a Pareto tail with tail inequality η given by

(1.14) $$\eta = \frac{\frac{\sigma^2}{2}}{\frac{\sigma^2}{2}+\mu} = \frac{1}{1+D}$$

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[13] Some of these results were originally presented in Stiglitz (1966). In the subsequent years, Pareto results have frequently emerged in models with heterogeneous agents. For a recent treatment, see Nirei and Aoki, 2014.

[14] Z is a standard Brownian motion, i.e. $dZ_t \equiv \lim_{\Delta t \to 0} \varepsilon_t \sqrt{\Delta t}$, where ε_t here is normally distributed with mean zero and unit variance.

[15] The use of diffusion models to describe inequality dates back to the work of Bevan (1974, 1979) and Bevan and Stiglitz (1979).

where

(1.15) $$D = \frac{\mu}{\left(\frac{\sigma^2}{2}\right)}$$

This gives the natural result that inequality decreases with the drift and increases with the variance of returns. One gets a result similar to that of Piketty (2014), seen more clearly if we rewrite

$$\mu = r(1-s) - (r-n).$$

The greater the difference between the rate of interest and the rate of growth n, the smaller the drift, and the larger the equilibrium level of inequality, *holding all else constant.* But r is an endogenous variable, and we need to relate it as well as the level of variability to the underlying parameters. Using the conditions for long run equilibrium,[16] $sf = nk^*$

(1.16) $$D^* = 2s^2 \ \frac{1-S_k}{S_k^2 n\bar{\sigma}^2}$$

In long run equilibrium, the tail-inequality *does not depend at all on the size of the difference between the rate of return and the rate of growth, but simply increases with the rate of growth and with the share of capital,* S_k. In the Solow model, n and s are taken as exogenous, but S_k is (except in the case of unitary elasticity production functions) endogenous.

Comparative statics: an increase in n

In analyzing the comparative statics effects, one has to ascertain the direct impact and the indirect impact, through the effect on the long run equilibrium. Thus, a very partial equilibrium analysis would note that an increase in the growth rate (n) leads to an increase in the "drift" and thus a reduction in inequality. But an increase in n has two further effects in the long run. It increases the rate of return on capital (since the equilibrium capital labor ratio is reduced), which has just the opposite effect on drift; but if, as we assume, the coefficient of variation in returns is constant, a higher rate of return is associated with more variability, and this leads to *more* inequality. (1.16) provides a simple formula showing how all of these effects get balanced off against each other.

Thus, an increase in n leads to less inequality, if factor shares remain the same. But an increase in n will normally change the factor distribution. An increase in n leads to an decrease in k, and hence an increase in the share of capital if the elasticity of substitution is less than unity. If the elasticity of

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[16] $D^* = 2(n - sr)/r^2 \bar{\sigma}^2 = 2n(1 - rk/f)/r^2 \bar{\sigma}^2 = 2s^2(1 - S_k)/S_k^2 n \bar{\sigma}^2$

substitution is less than unity, this reinforces the direct effect. There is thus a critical value of the elasticity of substitution $\varepsilon, \varepsilon^* > 1$, such that

$$\frac{\partial\eta}{\partial \mathrm{n}} > 0 \text{ or} < 0 \text{ as } \varepsilon^* \ < \text{or} > \ \varepsilon^*. \tag{1.17}$$

Provided the elasticity of substitution is not too great, an increase in the growth rate increases tail inequality.

Comparative statics: an increase in s

Similarly, a very partial equilibrium analysis would observe that an increase in the savings rate reinforces any differences in wealth that arise, i.e. the drift is reduced so inequality increases. But the fuller analysis provided by (1.16) notes that an increase in s leads to an increase in k, so

An increase in the savings rate leads to decreased inequality in wealth (in the tail) unless the elasticity of substitution is too great (with the critical value being in excess of unity.) [17]

Changes in the US and many other advanced countries associated with a decrease in both the savings rate and n would thus pull in different directions. But the observed increase in the share of capital would suggest (if sustained) an increase in inequality in wealth (in the tail) over the long run.

Similar results obtain if the sole source of variability in inheritance (per capita) is the result of variations in family size. Ignoring the (important) problems posed by the discreteness of family size and the limited range of family sizes, but assume family size is described by a standard stochastic process, with $\sigma = n\bar{\sigma}$, then we obtain

$$D^* = 2\frac{1-S_k}{n\bar{\sigma}^2}. \tag{1.18}$$

A decrease in n would thus again lead to a lower level of tail inequality, provided that the elasticity of substitution is not too much in excess of unity. Similarly, a decrease in the variability in family size (at any value of n) would lead, as expected, to less wealth inequality.

Capital Taxation[18]

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[17] Identical results hold if there is labor augmenting progress, in the analysis of the detrended distribution of wealth. We would then replace in the above equations n with $g = n + \lambda$.

A tax at the rate τ^c on the return to capital, with proceeds redistributed to workers leaves the aggregate capital accumulation equation and therefore the steady state unchanged, but increases the speed of convergence:

$$\frac{dlog(k_i)}{dt} - \frac{dlog(k_j)}{dt} = s(w + \tau^c rk)\left(\frac{1}{k_i} - \frac{1}{k_j}\right).$$

Now for higher income individuals,

$$\frac{dlog(k_i)}{dt} \approx sr(1 - \tau^c) - n,$$

and if we have stochastic returns to capital, as before, the Pareto coefficient will be described by the same equations as before, except now

$$D^* = 2s^2 \frac{1-(1-\tau^c)S_k}{S_k^2 n(1-\tau^c)^2 \bar{\sigma}^2},$$

where, it will be recalled, S_k is the before tax share. It is clear that the capital tax reduces tail-inequality.

1.4. 2. Variable wages [19]

Assume wages for each family are determined by the *same* stochastic process, with regression towards mean, that families optimize intergenerational utility and that there is a lower bound on wealth (individuals can't borrow more than a certain amount). The latter assumption turns out to play an important role in the determination of wealth and consumption inequalities.

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[18] For earlier treatments of taxation in models of equilibrium wealth distributions, see Stiglitz (1976b, 1978).

[19] The details of this model are set forth in Bevan and Stiglitz (1979). The notion of regression towards the mean is well-established, dating at least back to Galton (1886). See also Dewey (1889). As we explain below, there may be some forces weakening, and even reversing, regression towards the mean (referred to below as "trend reinforcement.) The Bevan-Stiglitz model, like the later work of Becker and Tomes (1979, 1986, 1994), entails endogenous decisions concerning bequests, where individuals assess the value of bequests in terms of impacts on later generations (a quite different approach from that of Piketty and Saez (2013) described below.) But unlike Becker and Tomes, we do not endogenize fertility decisions. And none of the studies fully incorporate interactions between human and financial capital and the investments in children that are publicly provided. On this, we side with Arthur Goldberg (1989) who argued that what he referred to as more "mechanical" models (for instance, with savings rate and fertility rates described by pre-specified stochastic processes) may provide more insights than those that try to endogenize all the relevant variables. Indeed, modern behavioral economics goes further: it suggests that the simple models may provide a more accurate description of the economy. Ironically, some of the principle "predictions" of the Becker-Tomes model which they argued showed the power of their model have subsequently been questioned. For instance, they suggest that "fertility is positively related to the wealth of parents" and that consumption "would not tend to regress at all among rich families who leave gifts and bequests to their children." More recently, Lindahl *et al* 2013) have provided an empirical test of the model on Swedish data, which rejects the model.

Then there exists an equilibrium wealth distribution which is related to the nature of the stochastic process of wages, the intertemporal/intergenerational discount factor, the interest rate, the degree of altruism across generations, and the elasticity of marginal utility.[20] Some of the effects are, however, quite complicated. A slow rate of regression towards the mean implies more wage inequality. Not surprisingly, if the wage dispersion is larger, wealth inequality will be greater. (Wage dispersion is also related to the dispersion in the wages of a child, given any particular level of wages of the parent.) At a fixed savings rate, a slower regression towards the mean implies more wealth inequality both because of the greater wage inequality and because of the compounding effect (high wages are likely to be followed by high wages.) This effect is even larger if the interest rate is high. But if there is slower regression towards the mean, then there is more need for those who are lucky (have high incomes) to save, to redistribute income from themselves to later generations. If the current generation saves more out of wages or inherited capital, either because it has more concern for future generations, or because there is a slower regression towards the mean, wealth inequality will be higher, reinforcing the direct effect of the slower regression towards the mean. [21]

But in this model, inheritances are designed to smooth consumption, and within the standard social welfare framework, it is inequality in consumption with which we should be concerned.[22] In the

[20] A slight modification of the model can be used to generate the earnings distribution: the model described above provides the dynamics of "skill" levels, which in turn can be translated into earnings differentials. (Of course, the earnings differentials associated with particular skill differentials may change over time.) Bevan (1979) argues that a model generating a simplified two-parameter version of the Champernowne (1953) wage distribution provides a good description of the wealth distribution, "performing markedly better than the lognormal." (The Champernowne distribution has cumulative form $F(y) = 1 - \frac{2}{\pi} tan^{-1} \left(\frac{u}{y}\right)^{\eta}$ where $F(y)$ is the proportion of individuals whose earnings are less than y, u is the median, and η is a constant, related to the coefficient of the Pareto tail.)

[21] The expression for the inequality of wealth provided by Bevan and Stiglitz (1979) is fairly complicated, but simplifies for the case where individuals save a given fraction, s, of their total lifetime income, dividing it equally among their children. If n and r are small, then $V_k^2 \approx \frac{V_w^2 s^2}{2(n-sr)(1-\beta+n-sr)}$, where β is the extent of regression towards the mean ($\beta = 1$ means that there is no regression towards the mean, $\beta = 0$ means that inheritance plays no role) and V_k and V_w are the variances in k and w, respectively. (Note that in this model, V_w is itself an endogenous variable.) Wealth inequality increases with the difference between sr and n. But, as we emphasized above, r is an endogenous variable, and one should relate the degree of inequality to the exogenous parameters of the model, as we did above. Moreover, in the more general model analyzed by Bevan (1974, 1979) and Bevan and Stiglitz (1979) s itself is an endogenous variable, affected by the underlying parameters. Thus, a lower level of β might lead to a higher savings rate, since parents with high wages know that their children are not going to do as well; this goes in the opposite direction of the *direct* effect of a decrease in β, which is to lower the variance of wages and, at a fixed value of V_w, to lower V_k.

[22] As we comment below, matters are somewhat more complicated. If individuals are concerned with their heirs, then it is inequality in dynastic utility with which we should be concerned. But there are reasons to be concerned with inequality in wealth itself--such inequalities have societal consequences that go beyond just the inequalities in

simplified model in which parents save a fixed fraction of their income and divide their wealth equally among their children, then the sign of the effects of an increase in the interest rate, the growth rate, and the rate of regression towards the mean on consumption inequality are the same as for wealth inequality. Because inequality is reduced with a decrease in the after tax return to capital, an inheritance tax (or a tax on capital) lowers the equilibrium degree of inequality, *assuming that the average savings rate is unchanged (*and, correspondingly, that the before tax rate of return on capital is unchanged). There is a presumption, however, that the savings rate will change. In the limiting case of a near 100% tax on inheritances, s would presumably fall close to zero, and for plausible values of the relevant parameters, inequality of consumption will increase.[23]

More generally, the effect of a higher rate of savings on consumption inequality is ambiguous; at a fast rate of regression towards the mean and a low interest rate, it reduces inequality; but at a low rate of regression towards the mean and a high rate of interest, it increases inequality.[24]

2. Kaldorian savings

In Kaldor's model, a given fraction of profits, s_p, are saved, and none of wages.[25]

(2.1) $\frac{d}{dt}(\log k_i) = s_{pi} r_i - n_i$,

where we continue to focus on the case where there is no labor augmenting technological progress, so that in the long run, $g = n$. It is easy to translate these results into the more general case.

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consumption and dynastic utility to which wealth inequality gives rise. This paper is just concerned with providing analytic models *describing* the inequalities in income, wealth and consumption. For a more extensive discussion of the normative issues, see Stiglitz (2012a, 1976a, 1976b, 1978), Bevan and Stiglitz (1979), and Kanbur and Stiglitz (2015). We also discuss below the implications for inheritance taxes.

[23] In the notation of our earlier footnote, for small r, $V_c(s) \approx (1-s)^2 V_w \frac{1+n-\beta}{1+n-\beta(1+sr)}$. With 100% inheritance taxes, $s = 0$ and $V_c = V_w >< V_c(s)$ as $s <$ or $> 2 - \beta r$. r is the return to capital over a generation, and is of the order of magnitude of 1, $0 < \beta < 1$, but standard estimate (see Bevan, 1979) suggest a plausible value for β is $.5$, which would imply that a 100% tax on inheritances would increase inequality of consumption provided s is less than 1 (which it obviously is.) This analysis ignores the feedback from the tax on the before tax return on capital.

[24] Moreover, even for small r, the variance of consumption does not depend simply on $sr - n$. It decreases in $n - \beta sr$, and increases with βsr. Again, however, we should relate r to the underlying parameters. Using the Solow model, $V_c^*(s) \approx (1-s)^2 V_w \frac{1+n-\beta}{1+n-\beta(1+sr)} = (1-s)^2 V_w \frac{1+n-\beta}{1+n-\beta(1+nS_k)}$. An increase in the growth rate leads to an increase in inequality of consumption, provided the elasticity of substitution is not too large (with the critical value being well in excess of unity); and an increase in the pace of regression towards the mean

[25] Similar results are obtained in the Pasinetti two-class savings model, a variant of which we present below in Part III of this paper. (Pasinetti 1962.)

Assume s_p, r, and n are the same for all families.[26] *Then relative wealth of all families would remain the same; any initial inequality of wealth would be perpetuated*, a result which contrasts starkly with that of the Solow model, where any initial differences in wealth asymptotically have no consequences[27]:

(2.2) $\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j) = 0.$

Note that this is true regardless of the value of r, n, or s_p—that is, the relationship between the interest rate and the growth rate has nothing to do with relative wealth inequality. *Contrary to Piketty, wealth inequality is unrelated to r - g.* In fact, however, in this case, in the long run there is a simple relationship between the rate of interest and the rate of growth: In long run equilibrium

(2.3) $s_p r = n$

so r is greater than rate of growth, but in spite of this, there is no *further* concentration of wealth. This is true even if $s_p = 1$.

Assume, on the other hand, that families differ in their savings rate, i.e. s_i for some family is greater than for some other family. Then its relative wealth will grow without bound. *There is ever increasing wealth concentration at the top.* This is a result which is consistent with Piketty's conclusions, but it arises not from the relationship between the rate of interest and the rate of growth[28] but from differences in the rate of savings among *different* capitalists.

As we noted earlier, any coherent model must reconcile macro-variables with the micro-analysis, i.e.

$$\frac{dK}{dt} = \sum_i \frac{dK_i}{dt} = \sum_i s_i r K_i = r \sum_i s_i K_i = r s^* K$$

where

$$s^* \equiv \sum_i s_i K_i / K,$$

is the weighted average savings rate. There is not only increasing wealth concentration at the top, but also an increasing average savings rate, an increasing capital labor and capital output ratio, and a diminishing rate of return on capital. But r, K/Y and s all approach asymptotically limiting values given

[26] It seems as if Piketty assumes $s_p = 1$ for all families, consistent with this assumption.

[27] This result was originally derived in Stiglitz (1969a). One of the motivations of this paper was to reconcile these results with those asserted by Piketty (2014) in a seemingly similar model.

[28] In this model, with no technological change, the rate of growth of population is equal to the rate of growth of the economy.

by r^{**}, $(K/Y)^{**}$, s^{**}, respectively. Then asymptotically all of wealth is in hands of the families with the highest savings rate, and

(2.4) $s^{**} = \max s_{i,}$

(2.5) $r^{**} = \frac{n}{s^{**}} = f'(k^{**})$

and

(2.6) $(K/Y)^{**} = f(k^{**})/k^{**}$

The long run equilibrium is dominated by the family (families) with highest value of s_i..

2.1. Taxation

This model illustrates well the risks of ignoring the possibility of tax-shifting.[29] Assume that a tax is imposed at the rate τ^c on the return to capital, with proceeds rebated to workers. We focus on the simple case where all capitalists have the same value of s_p, r, and n. Then the long run equilibrium is described by

(2.7) $s_p r(1 - \tau^c) = n$

The after tax return $r(1 - \tau^c)$ is unaffected by the tax. There is full shifting. But this means the capital labor ratio has decreased, and that workers' wages have decreased. To ascertain whether workers are better or worse off, we need to ascertain what happens to $w(k) + \tau^c f'(k)k$ as τ^c increases. It can be shown that this actually decreases: *workers net are worse off.*[30]

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[29] The central message of Stiglitz (1978) and Stiglitz (1976b) was that one had to be careful in the analysis of the incidence of capital taxation; under not implausible conditions, such taxation could lead to an increase in wealth inequality. The discussion here focuses on only one aspect of the long run general equilibrium analysis. See the discussion below.

[30] The proof is contained in Part III of this paper. The potential significance of these general equilibrium effects provides an important note of caution at other studies of optimal inheritance taxes in models which ignore these effects. See, e.g. Piketty and Saez, 2013. In their model, in addition, utility is defined over the size of the bequest, rather than as it would be in a more natural dynastic model, over the utility of descendants themselves, which would itself be affected by the bequests. Bevan and Stiglitz (1979) analyze bequests in such a framework, but did not analyze the implications for tax policy. (Moreover, the Bevan-Stiglitz model made other strong assumptions--everyone had the same dynastic preferences, and confronted the same wage generating stochastic process. An important contribution of Piketty and Saez was to incorporate heterogeneous preferences.)

There are important welfare consequences associated with these alternative approaches. In the Bevan-Stiglitz model, as we have noted, high wage workers, on average, have more wealth, which they are setting aside in the

The problem is that the transfer of money from capitalists to workers lowers average savings rates, and this leads to an increase in the return to capital, with a shifting of the burden of taxation.

If, instead, government invests the tax proceeds as well as the proceeds it gets from its investments, then an increasing fraction of the capital stock will be owned by the government

$$\frac{d}{dt}(\log K_g) - \frac{d}{dt}(\log K_p) = r\left(1 + \frac{\tau^c K_p}{K_g} - s_p(1-\tau^c)\right) > 0.$$

where K_g is the capital stock owned by the government, K_p is that of the private sector. The wealth of the capitalists can't keep up with the increase in population. Their wealth diminishes, and we get a new equilibrium which is similar to the original equilibrium except that now the government owns all the capital and, in effect, its saving rate is unity. Then wages are higher, and workers are unambiguously better off. Note that this would be true even if the government were slightly less efficient than the private sector.[31]

If we expand the model to a three factor production function, $Y = F(K_p, K_g, L)$, with private and public capital goods, and (some of) the proceeds from the tax are invested into the public capital good, then it is easy to show that there can be a new equilibrium in which a (somewhat poorer) capitalist class survives but the tax *may* still have a positive effect on workers: In a three factor production function, K_p and L can be substitutes, and K_g and L can be complements, so that on both accounts, wages are

knowledge that their descendants will be poorer. Within a utilitarian framework across generations, bequests are consumption and utility smoothing. The inheritance tax would be welfare Pareto enhancing from an ex ante expected utility perspective only to the extent that it improved risk sharing opportunities relative to the market--which it normally would, given constraints on borrowing; though obviously, from a utilitarian perspective, there would be gains from transferring wealth from those with good state variables today, i.e. from rich to the poor and from high wage families to low wage families.

In a framework in which utility is defined over bequests and consumption, as Bevan and Stiglitz point out, bequests are doubly blessed, because they enhance the utility of both the giver and the receiver, and this will be true even if as a result there is some (slight) increase in consumption inequality. There is an inherent externality, so it is obvious that the market solution will not be efficient.

There are, of course, other consequences of differences in access to resources, which arguably should play a first order effect in the design of optimal inheritance taxes.

[31] If the government invested only a fraction z of its revenues, then if z is small enough ($< s_p r(1-\tau^c) \equiv z^*$), there is an equilibrium ratio of $\frac{K_p}{K_g}$ given by $\frac{s_p(1-\tau^c)-z}{\tau^c z}$. For $z < z^*$, $k = f'^{-1}\left(\frac{n}{s_p(1-\tau^c)}\right)$. For a fixed τ^c, changes in z have no effect on the wages received by workers. The payments from the government (per worker) are $(1-z)r(k-(1-\tau^c)k_p)$. We already noted that at the limiting case where $z = 0$, workers are worse off than they would be without taxation.

increased as a result of the tax; but the increase in K_g is consistent with the after tax return to capital returning to its previous level.[32]

Progressive capital taxation

Assume, however, that we impose a *progressive* tax on capital, at an average rate of $\tau^c(k_i)$, $\tau' > 0$, so that the net return of a family with income k_i is $r\left(1 - \tau^c(k_i)\right)$. Without progressive taxation, if there are no differences in savings rates and everyone has the same return to capital, there is no convergence (equation 2.2), but with even slightly progressive capital taxes there is complete convergence:[33]

$$\frac{d}{dt}(\log k_i) - \frac{d}{dt}\left(\log k_j\right) = -sr\left(\tau^c(k_i) - \tau^c\left(k_j\right)\right) > \text{or} < 0 \text{ as } k_i < \text{or} > k_j \ .$$

The per capita wealth of the "poor" family increases relative to the rich. There is convergence *within capitalists*. But with convergence, we obtain the results described earlier: unless the proceeds of the tax are invested, the tax on capital raises the before tax return so much that workers are actually worse off *even if all the proceeds are rebated to workers*.

By the same token, regressive capital taxation will speed up the process of wealth divergence. De facto, it appears that the US has regressive capital taxation; those at the top are better able to take advantage of a variety of provisions in the tax code to lower their effective tax rate.

2.2. Savings and returns to capital among capitalists

This analysis has made it clear that what matters is the relation between sr and the growth rate, not r and the growth rate. The savings rate for even the rich is less than unity (especially once one accounts for consumption of housing). But to the extent that $s_p < 1$, the equilibrium return will be that much greater than the rate of growth.

[32] That is, the equilibrium is described by the solution to the pair of equations (in the natural notation)

(i) $(1 - \tau^c)s_p f_{k_p} = n$

(ii) $\frac{\tau^c f_{k_p} k_p}{k_g} + f_{k_g} = n$

The latter equation is derived by observing that $\frac{dk_g}{dt} = \tau^c f_{k_p} k_p + f_{k_g} k_g$

[33] Such taxes may have further general equilibrium effects, depending on how the proceeds of the taxes are spent. See the discussion below.

So far, we have assumed that the return to capital of all capitalists is the same. Assume that all families have the same savings rates and reproduction rates, but some families obtain a higher return to their capital. (There is a semantic question about whether one should view these higher returns as a return to a particular type of labor service—the ability to manage capital. In this view, "excess" returns to capital should be viewed not really as a return to capital, but as a return to labor. Nothing hinges on this semantic issue, other than the distribution of income among *factors.*[34]) Then

(2.7) $\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j) = s_p(r_i - r_j)$,

Again, capital gets increasingly concentrated in the family with the highest return.

2.3. Stochastic returns

If returns are random, with each family every period having an equal chance of high or low returns, then there is no equality of opportunity, no convergence *on average:*

$$E\left(\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j)\right) = 0.$$

Of course, some rich families have bad luck, have low returns, and their children begin life with poorer prospects than they did.

This is, however, a model in which the variance of per capita wealth amongst different families increases without bound, if the returns for each family are i.i.d. random variables. The problem is that in the long run equilibrium, $s_p r - n = 0$, so there is no drift ($\mu = 0$).[35] This property remains true, as we have noted, even if we impose a proportional tax on capitalists rebated to workers.

If, however, we impose a proportional tax on capitalists, rebated only to capitalists, then the income of the ith capitalist is $\tau^c r\bar{k} + (1 - \tau^c) r k_i$. Then, aggregate savings remains unchanged, so r doesn't change, and using our earlier diffusion model,

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[34] But for purposes of taxation, the distinction *is* very important, as the dispute about compensation for equity managers (covered interest) illustrates. Most of the seeming returns to capital can be viewed as (i) a return to the management of capital; (ii) a return to risk bearing; and (iii) a return to market power and other forms of exploitation. We elaborate on these issues in Part III of this paper.

[35] In our model, capitalists save a given fraction of their income. An alternative hypothesis is that consumption depends on wealth, so that $\frac{dlog(k_i)}{dt} = r - n - c$, where c is the fraction of wealth consumed. Macro-consistency requires $r = n + c$, presenting once again the problem of zero drift.

$$\mu = -\tau^c s_p r, \sigma = s_p(1-\tau^c) r\bar{\sigma},$$

from which it follows that the degree of inequality, as measured by the Pareto coefficient, decreases with the tax rate and increases with the rate of growth.[36] The pace of regression towards the mean will be even stronger if we impose a progressive capital tax, and so the level of inequality will be even lower.

If there is also variability in the rate of growth of the family size, then (2.7) becomes

(2.8) $\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j) = s_p(r_i - r_j) - (n_i - n_j)$.

In the limit, if the only source of variability is n, it is the variance of n that drives wealth inequality.[37]

2.4. Wealth begets wealth

If there is regression towards the mean in the ability to manage capital (as in the Bevan-Stiglitz model), then there will be an equilibrium wealth distribution with greater inequality, with equilibrium inequality greater the slower the speed of regression towards the mean and the greater the variance of abilities of the children, given the ability of the parent.[38]

As we noted in section 1, even without any hereditary link in the ability to manage capital, wealthier individuals may be able to obtain higher returns. We thus postulate that the return to capital for any family is a function of its capital,

(2.9) $r_i = r(k)\rho\left(\frac{k_i}{k}\right)$, with $\rho' > 0, \rho(1) = 1$.

In the formulation of (2.9) the only differences are those that arise out of the optimal amount of resources to allocate to the management of wealth, but it is straightforward to extend the model to the

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[36] We have made use of the long run equilibrium condition that $s_p r = n$. The result follows directly from recalling that $\eta = \frac{\frac{\sigma^2}{2}}{\frac{\sigma^2}{2}+\mu} = \frac{1}{1+D}$, where $D = \frac{\mu}{\frac{\sigma^2}{2}}$. Substituting the above values, we obtain $= \frac{2\tau^c}{n(1-\tau^c)^2\bar{\sigma}}$. .

Similar results obtain if part of the tax revenue is rebated to workers. What is remarkable about the above expression is that the savings rate of capitalists does not matter: the effects of an increase in s_p are fully offset by a corresponding change in the before tax returns.

[37] Under the natural specification, $\sigma = n\bar{\sigma}$--variability increases with the level of growth. Then $D = \frac{2\tau^c}{n\bar{\sigma}}$.

[38] This would be true if there were a fixed savings rate; but it is even more so because those with higher wealth will save more (to share some of their good fortune with their descendants, and because their returns to investing are likely to be higher.)

case where there is hereditability in the efficacy of management of capital. If that is the case, then (2.7) and (2.9) imply that families that are richer initially will increase in wealth relative to other families—and this is true regardless of the relationship between the (average) rate of return on capital and the rate of growth. As before, the economy comes to be dominated by the family (families) with the highest levels of wealth. The system is unstable, in that any perturbation, in which one family gets more wealth than another, is not only perpetuated, but the differences increase, to the point where a single family has all the wealth. [39]

On the other hand, if we combine a model with stochastic ability with regression towards the mean with increasing returns to capital, it is still possible that there be an equilibrium wealth distribution. The advantages of returns to scale are offset by the (eventual) disadvantage of relative incompetence (including the incompetence arising from not knowing that one is incompetent and the inability to hire competent managers). There is ample anecdotal evidence of this process at work. Still, the effects of the advantages of scale reflected in (2.9) can give rise to large inequalities. As before, progressive capital taxation can reduce these inequalities, regressive taxation (as in the US) can increase them.

It should be emphasized, however, that it is not inevitable that the stochastic processes exhibit regression towards the mean. It could exhibit what Battiston *et al* (2012) refer to as trend reinforcement, as those with greater wealth can get higher returns, not just because of the increasing returns properties of information (Radner and Stiglitz, 1984), but also because they can borrow at lower rates; they can use the power of their wealth to obtain greater rents, especially in the political sphere; they can better take advantage of tax loopholes, so their effective rate of taxation may be lower; and they may have access to inside information or other connections which yield them higher returns on their capital.[40] Thus, it should be clear that the increasing returns to capital ownership reflected in equation (2.9) are not necessarily *social* returns. It may only reflect the enhanced ability to grab rents.

2.5 Inherited human capital and progressive taxation

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[39] Sufficiently progressive capital taxation can undo the effects just noted.

If there are reduced (relative) returns for $\frac{k_i}{k}$ beyond a certain level, then all families with initial conditions with k_i above this critical level will see their per capita wealth diminish relative to those with less wealth.

[40] As we note below, there is an element of this not only at the top, but also at the bottom: those at the bottom who become indebted have to pay increasingly large interest rates as their net wealth diminishes.

Assume now that the savings out of (after tax) wages may be less than out of profits, and that there are separate taxes on wages and capital, with each being progressive, but that all families have the same innate abilities, savings rates, etc. We assume further that if an individual would have inherited a total wealth of k, that wealth would have been allocated between human and financial capital, according to $b(k)$. As individuals get wealthier, because of diminishing returns to investments in human capital, incremental wealth gets allocated to financial capital. We denote the after tax income functions by T^w and T^k respectively, that is $T^w(w)$ is the after tax wage income of someone with a wage of w. The rate of growth of the wealth of a family with wealth k is given by $H(k) - n$, where

$$(2.10)\ H(k) \equiv \frac{s_w T^w w\big(b(k)\big)}{k} + s_p T^k \frac{r\Big(k\big(1-b(k)\big)\Big)k\big(1-b(k)\big)}{k}$$

where s_w is the savings rate out of wages. If $H(k)$ is monotonically declining, as in Figure 2a, wealth will be equally distributed: families with k greater than k^*will see their wealth (per capita) diminish. But Figure 2b shows a case where, because of the increasing returns to investment, savings per capita divided by k may, beyond some k, increase. Then, inequality can increase without bound. Finally, Figure 2c shows a case where, because of the onset of strong progressive taxation--or the inability of people with excessive wealth to manage their wealth-- eventually $H(k)$ declines below n. Then, the economy would wind up with a bi-modal wealth distribution, at $k^*$and k^{**}. (Of course, in the more general case where there is inheritability of productivity, where inheritability is stochastic, and/or whether the returns to capital are stochastic, there can be more realistic wealth distributions.)[41]

2.6. Inequality of consumption versus inequality of wealth

Traditional social welfare focuses not on inequalities of wealth but of consumption, and in the models that we have presented, these do not translate simply. Consider the limiting case of the Kaldorian model where $s_p = 1$, and all workers have the same wages and save a fixed fraction of their income. Then, though there can be great wealth inequalities (and initial wealth inequalities among capitalists are perpetuated)[42], because the capitalists (by assumption) consume nothing, there are no consumption inequities. But if $s_p < 1$, then wealth inequalities translate directly into consumption inequalities.[43]

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[41] For a more extensive discussion of the implications of individual's allocation of wealth between financial and human capital (and the potential impacts of inheritance taxation on that allocation) see Stiglitz (1978).

[42] As before, wealth inequalities among workers vanish.

[43] Similarly, we noted above that in the Bevan-Stiglitz model with regression of wages towards the mean, where savings serves to smooth consumption intertemporally, reductions in these intertemporal transfers may lead to

In the concluding remarks, we comment on why, even in this model (which does not capture key elements of inequality in our society today) we should, nonetheless be concerned with wealth inequality.

less wealth inequality, but greater consumption inequality. For a more extensive discussion of this point, see Stiglitz (1976b).

3. The centrifugal and centripetal forces in the economy

The above models provide a framework for understanding the forces that can lead to an equilibrium wealth distribution that is more or less unequal. As we have already noted, differences in $w_i s_i, r_i$, and n_i and the stochastic processes for these variables determine differences in relative wealth positions. Indeed, the model presented in the beginning of section 1 in which there is no regression towards the mean in wages can be thought of as one limiting case. The other extreme is that where there is no correlation across generations (effectively, equal opportunity.) There will still be inequality in wealth and consumption.

More dispersion of returns to capital and wages and persistence of differences in returns and wages will lead to more dispersion of wealth. Thus, one of the factors limiting the build-up of ever increasing inequality is that the children and grandchildren of those who made a fortune typically have a greater likelihood of squandering the family fortune than magnifying it, reflected in the adage, from rags, to riches, and back to rags in three generations.

So too, as we have noted, more dispersion in reproductive rates and savings rates will lead to greater dispersion in wealth. If some families have a small number of children, inheritances will have to be divided among a smaller number, and so wealth (per capita) increases. In particular, if richer families have smaller families, then there will be more wealth inequality.[44]

Differences in norms and social custom can lead to other aspects of wealth dispersion: if some groups in society save more, than they will have more wealth. But one has to be careful in interpreting observed differences: as we noted earlier, it is possible that there is a poverty trap. Those with low income and capital save little, but if one could somehow increase their income and wealth enough, they would move out of this poverty trap. It is not that differences in savings caused the observed differences in income; it is that the observed differences in income and wealth caused differences in savings rates. This may be especially so if, as we have suggested, the returns to capital are higher for those with more wealth; for then, their incentive to save may be higher. (As we showed in the previous sections, even the basic model can give rise to wealth inequality.[45])

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[44] But the boundary value condition, where there are no children, has just the opposite effect, if those without children give their estate away. Similar results hold if wealthy individuals decide not to give a large proportion of their wealth to their children.

[45] Assume the savings function of each family is a function of k_i. Then, instead of (1 .6), we have

3.1 Beyond the basic model

The model helps frame a discussion of other forces that could lead to greater inequality, by focusing on the underlying drivers, the level of dispersion of wages and returns to capital, the perpetuation of those differences, and the transmission of advantages and disadvantages across generations.

Inheritance

For instance, if instead of dividing inheritances equally, the eldest son inherits all the wealth (primogeniture), then there will be more inequality. Changes in norms of inheritance—where it becomes more the norm to divide wealth equally—lead to more equality of wealth. (Changes in norms are often translated into or accelerated by changes in laws, e.g. prohibiting primogeniture .)[46]

Demographics

We noticed the potential role of demographics. There are other ways in which demographics, broadly understood, affect wealth inequality. If, for some reason, there was an increase in assortive mating, with those with high wages (productivities) marrying others with high wages, then arguably, the pace of regression towards the mean might be slowed. If previously, alpha males chose mates based on looks, rather than on characteristics that drove market returns, then the pace of regression towards the mean in wages would presumably be faster. If one organizes tertiary education to increase the likelihood of assortive mating on the basis of market productivity, then the pace of regression towards the mean will be slower.

The OECD (2011) has called attention to the role of changing social patterns (e.g. in family structure and household formation) on income inequality among households. These patterns translate, over time, into wealth inequalities. For instance, if there are some households with two earners and only one

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$\frac{d}{dt}(\log k_i) - \frac{d}{dt}(\log k_j) = w\left(\frac{s(k_i)}{k_i} - \frac{s(k_j)}{k_j}\right) + r\left(s(k_i) - s(k_j)\right)$. For the existence of multiple equilibria, we require s' to be sufficiently large: $\frac{dlog(s)}{dlog(k)} > (1 - S_k)$.

For a slightly fuller elaboration of such a model, see Part III of this paper.

[46] Stiglitz (1966) shows how a model of inheritance with primogeniture will give rise to a wealth distribution which is Pareto in the tails.

child, they are likely to pass on to their child more wealth than is the case for households with two or more children and only one earner. Patterns, such as those emerging in the United States, where those at the bottom of the distribution are less likely to get married are likely associated with less transmission of human capital across generations at the bottom, leading to more wealth and income inequality.[47]

Exploitation

In Part I of this paper, we suggested that some, perhaps much of the increase in wealth and wealth inequality is associated with an increase in exploitation, broadly understood. This can be affected not only by market power, but by norms and laws, and these often interact: market power gets translated into political power which leads to changes in laws. The moral deprivation that seemed so evident in the financial sector, especially in the years before the crisis, represented a change in norms that may have been facilitated by changes in laws and market power. These changes enabled those in that sector to engage in these practices, which often generated very high returns, with large distributive consequences in creating and perpetuating poverty at the bottom and riches at the top.[48] [49]

Intergenerational transmission of advantage

If the very rich can use their wealth, and more broadly the position in society that that wealth gives to them, to get higher returns to their capital and access to better jobs for their children ("rents" in the labor market, above normal returns in the capital market), then wealth will become more concentrated. This has, of course, always been true—connections matter, and connections are passed on across generations; but if the extent to which this is true changes, then there will be a change in the equilibrium distribution of income and wealth.

One might have thought that in a meritocratic society these connections would matter less, and that may indeed be the case in countries, like those in Scandinavia, which take meritocracy seriously. But in countries like the US, there is little evidence that the importance of connections has significantly decreased. Indeed, ironically, in an imperfect meritocracy, the importance of connections may actually be increased. For instance, increasingly to get a good job one needs an internship, which is often

[47] Greenstone and Looney (2012).
[48] See Stiglitz (2010a)
[49] In our model, it was assumed all workers received the value of their marginal products. But as we noted in Part I of this paper, it may be that those at the bottom are paid less than the value of their marginal product, and a part of the compensation of those at the top are rents that they receive related to their position.

unpaid. Not only can the children of the less affluent not afford these internships, but it often takes connections to even get this unpaid work.[50]

Connections matter in another sphere: politics. In many countries, those with connections are able to extract rents from the public.[51] This is true even in democracies, though it has to be done in a more "rule based" way: the manner in which the banks first "purchased" deregulation, and then received mega-bailouts, is a case in point.[52]

How wealth begets wealth—and how those in poverty become trapped there—is well understood. Those near bankruptcy have to pay higher interest rates, making their descent towards the bottom even more steep.[53] Their attempts at survival occupy so much of their energies that they cannot think about the long term; and accordingly, they do not make the long term investments that would increase their incomes.[54]

Those without wealth cannot get access to credit markets. This becomes especially important in an era of super low interest rates. But in an era in which interest rates are near zero—and even the return to many risky assets is very low—how can the inequality of income and wealth increase? Our usual models differentiate between "labor" and "capital" and, with the "savings glut," it would seem that the return to capital should have plummeted.[55] Shouldn't that mean that the share of capital would have plummeted too[56], and so too income and wealth inequality? In Piketty's analysis, this period of low interest rates should be an era of wealth convergence. But instead, there is wealth divergence. None of this has occurred, and the reasons that it hasn't are instructive.

Knowledge and inequality

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[50] Perlin (2011).

[51] Much of inequality in many countries is related to privatizations and the sale of public assets at below market prices. India's spectrum auction is one of the most recent examples. But there are many others. Sometimes the transfers occur in a more indirect way: the government issues a banking license to someone that is politically connected; the "private" bank lends money to favored parties to purchase the state assets that are being privatized. Restrictions on who can bid ensure that the prices are below what they would be in a competitive market. Much of the Russian oligarchy was created in this way.

To the extent that connections can be purchased, this just reinforces the increasing (private) returns associated with wealth ownership.

[52] See Johnson, Simon, and Kwak (2010) or Stiglitz (2010a).

[53] Battiston *et al.* (2012) refer to this as trend reinforcement.

[54] Mani *et al.* (2013); Mullainathan and Shafir (2009).

[55] Bernanke (2005).

[56] Under the assumption of an elasticity of substitution less than unity. See Part I of this paper for a discussion of the elasticity of substitution.

The scarce factor in our economy would seem not to be capital, but knowledge. Capital flows relatively freely across borders; yet differences in per capita income persist, and largely because of impediments to the free flow of knowledge. The banks' manipulation of the LIBOR and foreign exchange markets as well as insider trading scandals exemplify the returns that can be obtained from information asymmetries[57]—even information asymmetries deliberately created by the market. While these were *outside the law*, there are pervasive opportunities to do similar things (with perhaps slightly lower returns) *within the law.* It is the belief that there are returns to knowledge that motivates those who manage capital to invest so much in the acquisition of knowledge, and to work so hard to keep what they know secret.[58] But not everyone has equal access to knowledge; and in markets timing is critical: knowing something slightly before others can yield large (private) returns.[59]

Risk taking

Given the asymmetries of information—those without access to special information know the equity markets can be a stacked game—and given that less well-off individuals are more risk averse[60], it is natural that the richest individuals own a disproportionate share of equities; and if equities have a higher return than safer assets, then, on average, those at the top will see their wealth grow on average faster than those lower down. Moreover, as wealth increases, individual's ability to absorb risk increases. This means that as society gets wealthier, the dispersion of returns may increase—leading to fatter tail wealth distributions at the top. (This affect could be partially offset by improvements in the management of risk, so that the overall portfolio risk—which is what matters for the evolution of wealth inequality—is reduced. But these improvements would in turn lead to a still further increase in overall risk taking. The presumption is that the net amount of risk taking would still increase with wealth.)

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[57] It should be noted that only the most egregious examples of the use of inside information are illegal and get prosecuted.

[58] As it is sometimes put, "knowledge" is both power and money. For a broader discussion, see Greenwald and Stiglitz, 2014.

Interestingly, the efficient markets hypothesis suggested such investments yielded no return: information disseminated perfectly and instantaneously throughout the economy. But why then would rational individuals invest so much money in gathering information? See Grossman and Stiglitz (1980). The evidence, however, is that markets are not informationally efficient, and that means there are returns to investment in information/knowledge. See Shiller (2002).

[59] Especially if other market participants are overconfident or unaware of their informational disadvantage. Note again these are private returns, not social returns. See Stiglitz (1982). Differential access to technology and information processing abilities has similarly given rise to rents in equity markets, especially more recently, in high frequency trading. See Stiglitz (2014c) and the references cited there.

[60] It is a standard assumption that there is decreasing absolute risk aversion.

Education as a mechanism for the transmission of advantage

Earlier, we explained how, if richer individuals (high wage individuals) invest more in the human capital of their children, so their children have higher wages, the pace of regression towards mean will be slowed and there will be more wealth inequality.

High quality public education can counter this force, ensuring that everyone faces a more level playing field. If the educational system did this, it would be the most important centripetal force for equality in our society.

But in a society, like the US, where there is a reliance on local funding for schools, if there is more economic segregation,[61] then there will be more inequality in the transmission of human capital. So too if greater reliance is placed on tuition for financing tertiary education, in the absence of adequate scholarships; and this is even true if debt financing is made available, unless the debt repayments are income contingent, as in Australia. Higher interest rates charged on student loans will lead to more inequality of human capital; so too would the passage of a bankruptcy law that makes student debt not-dischargeable even in bankruptcy (as the US has done with a series of laws dating to the 1970s, the most recent expansion of which was the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005).

Changes in markets[62]

Changes in markets may also lead to changes in the equilibrium wealth distribution. Better insurance and annuity markets mean that individuals have to accumulate less precautionary and retirement savings. There is large variability in the time of death, and those who die early with large amounts of precautionary and retirement savings leave more to their children.[63] Better rental markets or reverse mortgages mean that the elderly are less likely to hold large amounts of real estate wealth--passing on less to their heirs upon death.[64] An increase in the difference between life expectancy and the age of

[61] Evidence is that economic segregation has increased. See Bischoff and Reardon (2011).
[62] We focus in this section on changes in markets other than the widely discussed ones affecting wage inequality, such as skill biased technological change, globalization, etc.
[63] These are sometimes referred to as "unplanned bequests," but that is not quite an accurate description: individuals should take this risk into account in their savings decisions. For early discussions of equilibrium wealth distributions arising from such bequests, see Stiglitz (1978) and Flemming (1979).
[64] It is worth noting that there are large differences across countries in the relative role of rental markets vs. home ownership. In Germany, homeownership is relatively low.

retirement[65] and an increase in the variance of the age of death will lead to more wealth inequality. Public social insurance programs (Medicare and social security) mean that individuals would have to hold less wealth against the risk that they live a long time, and hence imply less inequality.

Stiglitz (1978) constructs a simple model of stochastic death which gives rise to a Pareto tail, which is consistent with the above observations. He notes that since capital taxation increases the amount that individuals have to save for their retirement, it can lead to higher levels of average bequests and wealth inequality. With strong public social security programs, with the tax exemption of most life-cycle savings, and with defined benefit retirement programs, this effect is probably not significant; but with the weakening of public programs and a shift from defined benefit to defined contribution retirement programs, this effect could become more significant for those in the upper-middle incomes in the future.

Changes in public policy

Public policy can have, indirectly, a major impact on each aspect of the generation of wealth inequality described above. Here, I note only five points. First, following on our discussion of annuities, we note that the provision of public annuities reduces the need for individuals to save for retirement. But since most countries only provide limited public annuities, there are differential effects across the income distribution: It partially accounts for the essentially zero savings for retirement for the bottom part of the population. Hence, overall, wealth inequality (which is traditionally measured excluding implicit social security wealth) is probably increased. (While social security may have as a result increased wealth inequality, it markedly reduced consumption inequality.)

Secondly, taxation of capital and especially bequests has both an income and a substitution effect reducing bequests, and thus the transmission of inequality. A lowering of the tax on capital would be expected, in the "basic model" to lead to an increase in wealth inequality. Thus, the marked lowering since 1980 in these taxes may have played a significant role in the increase in wealth and income inequality.[66]

[65] This variable is critical in the life cycle model described in Part III of this paper. In the absence of annuity markets, individuals care not just about the mean life expectancy; the variability in life expectancy will also affect savings rates—and therefore the importance of life cycle savings.
[66] There is an important caveat to this conclusion, in at least some of the models we have examined. In, for instance, the Kaldorian model, there is full shifting; thus, the lowering of the tax rate simply leads, in equilibrium, to a lowering of the before tax return--with the after tax return unaffected.

Progressive capital taxes reduce wealth inequality, as we have seen. Changes in taxation in the United States have reduced progressivity. Indeed, today those with very high incomes pay much lower effective tax rates on their income than those with lower incomes. In the models explored here, this can in fact give rise to an ever increasing level of wealth inequality.

Thirdly, monetary policy, whether intentionally or not, affects the distribution of income and wealth. Quantitative easing increased the wealth of the wealthy individuals who own the bulk of equities. Low interest rates encourage firms to use more capital intensive technologies, reducing the demand especially for low skilled workers. If monetary authorities tighten whenever wages start to rise, the effect will be a ratcheting down of the wage share.[67]

Traditionally, the central distributional conflict confronting monetary authorities has been seen as that between debtors and creditors, with low interest rates benefiting the former at the expense of the latter. In Part IV of this paper, we show today that today, the conflict is often between owners of equity and owners of short term debt. The impact on wealth distribution may be driven by differences in portfolios.

Public policy affects the relative returns to different classes of assets and the riskiness of these assets; and in doing so affects the ownership distribution of the assets. Preferential treatment of capital gains taxes is of most value to the rich, and hence this tax policy not only benefits the rich, but also may lead to greater disparity in ownership patterns. Limitations on loss offsets may be less binding on high wealth investors, and hence these provisions may similarly have asymmetric effects.

Fourth, we note that any change in markets or public policy which affects the distribution of wages will (according to our basic model) affect over time the distribution of wealth. There is an extensive recent literature on the determinants of wage dispersion, discussing, for instance, how globalization and skill biased technological change may have led to greater wage inequality. But the extent to which this is true is not just determined by market forces, but how those market forces are shaped by public policy, e.g. the rules governing unionization and globalization.

Finally, we note that changes in policy affect not just the distribution of wages *among* workers, but also the distribution of factor incomes *between* workers and capitalists. For instance, asymmetric trade liberalization (where capital market and goods market liberalization precede labor market liberalization)

[67] See chapter 10 of Stiglitz (2012a) and Stiglitz (2015) for a more extensive discussion of the distributional effects of monetary policy.

exacerbates downward wage pressures in advanced countries. (Stiglitz and Charlton, 2005). Going forward, changes in the economy and in globalization, including the rules governing it, may affect inequality for another reason that we noted briefly above: the increasing share of services (Greenwald and Kahn, 2009) may increase the importance of local monopolies.

Cyclical effects

The models of this paper are concerned with the long run evolution of the wealth distribution. Yet, one cannot separate the consequences of economic instability from the long run analysis, particularly in the presence of asymmetries and hysteresis effects. It is those at the bottom that suffer the most from economic fluctuations (see, e.g. Furman and Stiglitz, 1998)), and in the boom, they do not make up for what they lose in the recession (especially if monetary authorities follow the kinds of policies described earlier). Instability may thus contribute to income and wealth inequality—the recent economic downturn being a case in point.[68] The extent to which this is so depends, of course, on both the strength and design of automatic stabilizers, like unemployment insurance, but also the strength and design of discretionary policies. (Policies, such as undertaken in the US in the aftermath of the 2008 crisis, which bailed out banks but did little to help homeowners contributed to the increase of inequality generated by that recession.)

An overview of the changing balance between centrifugal and centripetal forces in the economy

Three of the key centripetal forces in the economy may have weakened in recent decades, especially in the United States: the tendency for smaller families has weakened the effects of division among heirs; the reduction of progressivity of the tax system—to the point where at the upper reaches it has become regressive—may have changed the stochastic process describing returns from one characterized by mean reversion to one characterized by trend reinforcement; and the equalizing effect of public education has been weakened with increased economic segregation and increasing disparities between schools attended by the children of the rich and that of the poor. Meanwhile, some of the centrifugal forces may have become stronger—wage disparities have increased, with stagnation, or even decreases, in real wages of those at the bottom and soaring increases at the top; assortive mating combined with

[68] It should be pointed out, however, that these effects are not unambiguous, since many economic fluctuations are associated with stock market crashes that especially adversely affect those at the top. Income and wealth inequality fell after the stock market crash of 1929. The current crisis may have especially adversely affected workers because of the disproportionate effect on housing wealth, and government policies which seem to have restored stock market wealth more effectively than housing wealth.

greater female labor force participation has led to an increasing divide between families with two high income earners and at most one child, and those with one breadwinner, often working at low wage jobs; differentials in access to health care between the top and the bottom are one factor contributing to large observed differences in health status, reinforcing earnings differentials; and an increased scope for rent seeking noted in Part I of this paper too may have contributed to increases in incomes at the top.

Given all of this, it is not surprising that there has been increased disparity in the income and wealth distribution.

4. Concluding Comments

The models presented in this paper help focus on a critical question in today's society: the transmission of advantages across generations.[69]

The intergeneration transmission of advantage, whether accomplished through transfer of financial wealth, human capital, or "connections", is important because societies in which positions are in an essential way based on inheritance are fundamentally different from those in which positions arise from individual's own efforts and abilities. Such societies cannot claim that there is a level playing field, that there is equality of opportunity. There is already ample evidence that this is true in the US, with the children of the rich who perform poorly in school ending up with higher incomes than the children of the poor who do well; and with a young American's life prospects being heavily dependent on the income and education of his parents.

For more than two centuries, there has been an attempt to break away from a feudal system in which a child's position in society is pre-ordained by that of his parent, and move to a meritocratic system where it is determined by the child's own ability. In many respects we have succeeded, but perhaps not as much as we had hoped: the evidence is that even in a society like the United States avowedly committed to meritocracy, inherited advantages play a key role, and more than a role than can be

[69] While our models make simple and we believe plausible hypotheses about the savings behavior ofindividuals, it should be clear that similar results would hold in a model in which capitalists maximize their intertemporal dynastic utilities, and workers maximize their life-time utilities. Indeed, the model may even be consistent with one in which all workers maximize dynastic utilities, but for those workers with low wages and capital, the borrowing constraint is binding: in the absence of such constraints they would like to pass on negative wealth to their heirs, but given the constraint, they best they can do is to ignore them, and maximize their own utility. Adding such complexity to the model will, however, provide few additional insights.

explained by the process of transmission of genes. The models presented here help explain why that is so.

We should also be concerned with wealth inequality, however it is generated, because societies in which there are large wealth (and income) inequalities function differently from more equalitarian societies. There are social and political consequences. It is worth noting that the attack on monopolies and trusts in the Progressive era was more motivated by concerns about their political and social consequences than the market distortions to which they gave rise.

Throughout our analysis we have emphasized the importance of taking a general equilibrium perspective, with full consistency between the macro- and the microeconomics. Thus, we showed that in Solow model, the relevant Piketty condition $sr > g$, where $1 \geq s > 0$, was *never* satisfied in the long run; while the Kaldor model represented a knife-edge where $sr = g$. But in the absence of progressive capital taxation or some diminishing returns to wealth, with stochastic returns the Kaldor model generates ever increasing wealth inequality, increasing at a rate and a manner which seems inconsistent with what is usually observed.

When there is an equilibrium wealth distribution, we have been able to derive simple closed form expressions for the level of tail-inequality in terms of the underlying properties of household behavior (savings and reproduction) and technology (most importantly, relative *shares* and the variability in returns). This has allowed us to derive precise results about the effect of different forms of taxation on tail-inequality. Some policies that might seem to reduce inequality may, because of the shifting of taxes and expenditures, have a more ambiguous effect. For example, we showed that a tax on the return to capital, with the proceeds provided as payments to workers—a policy which on the face of it would seem to unambiguously reduce inequality—in the Kaldor model may have the opposite effect because of tax shifting; but if the proceeds of the tax are spent on public investment goods, there can be unambiguous reductions in inequality.[70] By contrast, in the Solow model, capital taxation redistributed to workers always lowers tail-inequality. And in the Kaldor model, *progressive* capital taxation reduces tail inequality when it is redistributed to less wealthy capitalists, so long as the *average* savings rate is not reduced.

[70] By contrast, taxes on the return to land (discussed in Part III of this paper), including capital gains, reduce wealth inequality and, under certain conditions, even lead workers to be better off.

Here, we have focused on the distribution of wealth among individuals, when all individuals follow the same savings behavior, or among capitalists, when it is they who are responsible for society's savings. But there has been increasing attention on life cycle savings, and the relative importance of life cycle savings to inherited wealth. This is the question to which we turn in Part III.

The models presented here help us understand how changes in the underlying key parameters, such as savings rates, bequest behavior, and reproduction rates, and the differences among families with respect to these variables affects the equilibrium wealth distribution. Throughout our analysis, we have also emphasized the key role of policies in determining inequality, and we have delineated in particular the effect of changes in taxes on the equilibrium level of inequality. Just as we argued in Part I of this paper that one could not explain the increase in the wealth-income ratio solely within a neoclassical model (without rents), so too one cannot fully explain the increase in wealth inequality within a neoclassical model (without rents), such as we have presented here. Part IV of this paper will explore in greater detail some of the determinants of the magnitude of rents in some very simple models.

Nor do I believe that will be able to account well for changes in inequality solely in terms of changes in the key variables identified above in the absence of *policies* which have affected those variables. Section 3 described how many of the policy changes in recent years had strengthened the centrifugal forces and weakened the centripetal forces. But that raises a deeper question: why have we adopted such policies? Thus, in the end, to understand the growing inequality in America and other advanced countries, one has to come to terms with *politics,* and how economic inequality translates into political inequality, which in turn leads to policies which reinforce the economic inequalities.

Because so much of the increase in inequality in income and wealth is related to changes in policies, changes in those policies may be able to ameliorate this growing inequality. If we believe that there are large costs to our economy, our democracies, and our societies of this growing inequality, then at the very least, we should ask, are there changes in policy which will slow down this increase in inequality, and perhaps reverse it. An understanding of the forces that may be contributing to the growing inequality, such as that we have attempted to provide here, is a first step in constructing such a policy agenda.

In fact, a long list of policy changes—changes in legal frameworks, taxes, and expenditures— which would lead to less inequality in both the short run and the long which might do this, and simultaneously increase economic performance, has already been identified.[71] It is not the lack of knowledge that is preventing these actions from being undertaken. It is politics, a politics shaped by inequality of political power which follows from and can amplify inequalities in economic power.[72] The growing inequality in our society is thus a reflection as much of democracy in the 21st century as it is of capitalism in the 21st century.

[71] See, e.g. Piketty (2014) and Stiglitz (2012b). Such changes affect both the distribution of income and wealth at any moment of time as well as the dynamics that describe the evolution of those variables. This paper has taken technology as exogenous, but as Braverman and Stiglitz (1989) point out, technology and technological change itself is affected by societal inequalities. Sharecropping is a prevalent tenancy arrangements in economies with large disparities in land ownership, but not otherwise. But the choice of technology at one moment affects the distribution of income and wealth and wealth dynamics, and even the nature of technological change (Greenwald and Stiglitz, 2014).

[72] The points raised here (and similar points made elsewhere in this paper) are echoed in Suresh Naidu's excellent review of Piketty (2014).

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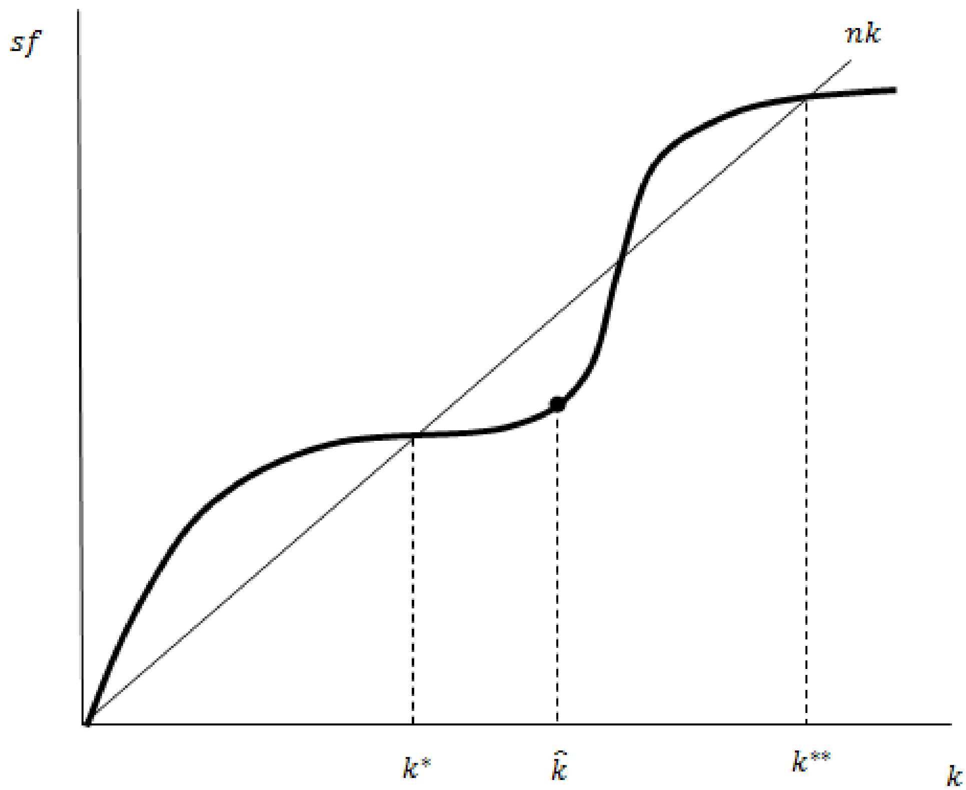

Figure 1

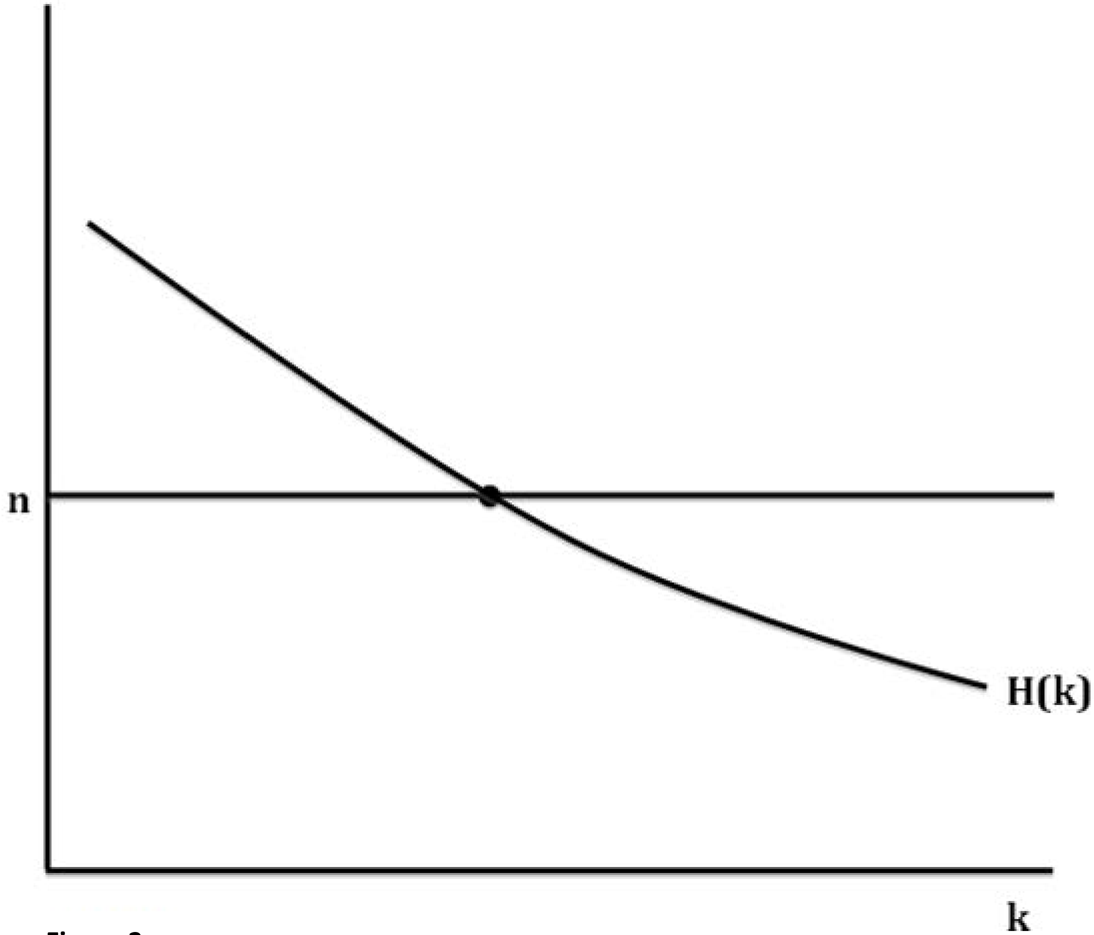

Figure 2a

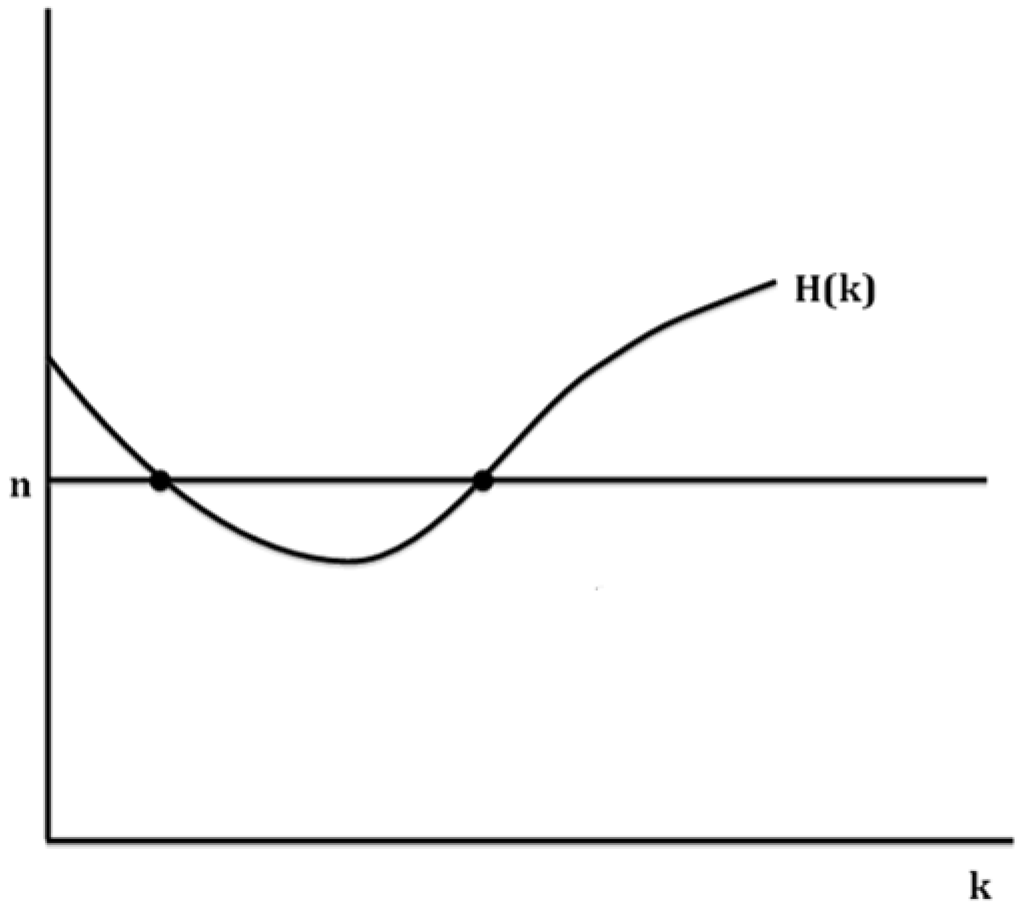

Figure 2b

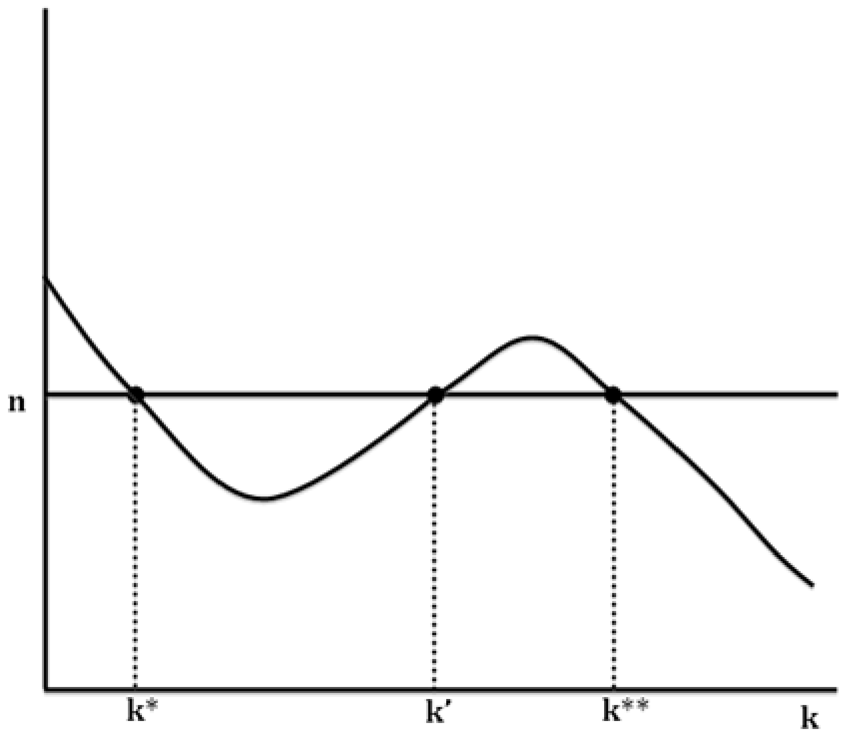

Figure 2c